

Chance and Skill in Poker

Professor Abraham J. Wyner
April 17th, 2008

I. Biography of the Author

Professor Abraham (Adi) Wyner is Associate Professor of Statistics at the Wharton School of Business. He came to Wharton in 1999, from the University of California at Berkeley, where he was an Assistant Professor and a NSF Post-Doctoral Fellow. Dr. Wyner did his B.S. in Mathematics at Yale University where he won the Stanley Prize for excellence in Mathematics, heading west to complete his doctorate in Statistics on the West Coast, at Stanford University.

Professor Wyner's principle focus at Wharton has been research in Applied Probability, Information Theory and Statistical Learning. He has published more than 20 articles in leading journals in many different fields, including Statistics, Probability, Information Theory, Computer Science and Bio-Informatics. He has received many grants from the NSF, NIH and private industry. Professor Wyner has participated in numerous consulting projects in various businesses.

He was one the earliest consultants for TiVo, Inc, where he helped to develop personalization software. Dr. Wyner created some of the first on-line data summarization tools, while acting as CTO for Surfnotes, Inc. More recently, he has developed statistical analyses for banks and marketing research firms and has served as consultant to several law firms in Philadelphia, New York and Washington, D.C. In addition, he has served as statistical faculty advisor for the University Pennsylvania Law School. His interest in sports statistics has led to an ongoing collaboration with ESPN.com and "ESPN: the Magazine" where Dr. Wyner is the PI on the ESPN funded MLB player evaluation research project. He has served as faculty advisor to the Wharton Quant Club, numerous MBA cohorts and the Wharton Gaming club. For several years he taught an undergraduate course in Gaming that was so popular that over 1000 students competed for only 12 slots.

II. Is Poker predominantly a game of Skill?

In this consultation, I will address the question of whether poker (and more specifically Texas Hold'em Poker) is a game whose outcome is dependent more on skill than on chance, by evaluating two scientific articles where the issue has been analyzed in detail. One is an article by Professor Noga Alon, of Tel Aviv University (which is attached to this opinion as Annex A) and a second is essentially a follow up to Alon's article, written by Laure Elie and Romuald Elie of the University of Paris (which is attached to this opinion as Annex B). They have applied mathematical techniques to provide scientific evidence to the fact that poker is a game wherein winning is more dependent on skill than on chance.

III. *Poker, Chance and Skill*, by Professor Noga Alon:

Noga Alon considers the game of “Texas Hold’Em” for which he provides a detailed and accurate description. Then he calculates probabilities for each type of hand and explains how knowledge of these probabilities is necessary in order to wager in a way that will maximize the expected winnings. This is his first intimation that a skilled player, who is able to calculate probabilities and use those calculations, will have an advantage over a player who cannot. One course of action, rejected by Alon, is to attempt to mathematically quantify the level of skill in a game. Instead, Alon constructs a simplified game of Texas Hold ‘Em poker which he uses a model. The basic argument is that of *a fortiori*:

if it is possible to demonstrate that Skill predominates in simplified Texas Hold ‘Em, than all the more so it will dominate for the actual game.

Alon constructs several different single-betting-round games based on a “basic game” constructed to depend on the ranking property of poker. The games are as follows:

1. A two player game involving a beginner “Bob” who plays randomly against an advanced “Alice,” who plays optimally.
2. An extension of the previous involving an “Improved” Bob against “Adapted” Alice, who adapts her advanced strategy to counter Bob.
3. A multiple player extension with Advanced Alice against multiple beginners in a ring game.

Alon shows that it is possible in these simple games to calculate exactly the strategy that Alice should play in order to maximize her expected winnings per round. Alon finds such a strategy for all three versions and then he calculates Alice’s expected winnings per round and the variance of her winnings. He then applies the Central Limit Theorem for repeated independent events to calculate (for version 1) the approximate chance that Alice does not have more money than Bob after n rounds of play. As an added twist, he calculates the same probability with a blind bet instead.

For the simplest version, Alon shows that Alice’s skill will dominate Bob’s luck based approach. In fact, we have that:

- Although 7 out of 8 games end in a draw, if there is a winner, then Advanced Alice is 3 times more likely to win than Beginner Bob
- After 15 rounds of play, the chance that Alice is ahead is about 84%.
- After 150 rounds, the chance that Alice is ahead is about 99.9%.

To summarize, Alon accurately argues that:

- Knowledge of hand probabilities is a learned skill fundamental to determining and implementing an advanced strategy
- An advanced strategy will earn more than a strategy of an unskilled player with high probability in the short run
- An advanced strategy will earn more than a strategy of an unskilled player in the long run, with certainty.

So it is abundantly clear that in a simple game which pits an expert against a novice¹, the skilled player will dominate quickly. **Skill is the deciding and dominant factor.**

IV. Limitations and Extensions: *Chance and Strategy in Poker*. By Laure Elie and Romuald Elie.

The Alon analysis is of course limited to a basic simplified one round game of pseudo-poker. To conclude that poker itself is predominantly skill, one has to accept that the intricacies of actual poker will necessarily favor the skill factor, from which it follows, *a fortiori*, that real poker is predominantly skill. The argument is a heuristic, but it is compelling.

A second limitation is Alon's choice of players where Bob, who plays with basically no skill at all, challenges advanced player Alice. A more convincing argument would show that skill dominates the outcome of a game involving a highly skilled opponent against a player of modest abilities.

This challenge is met by the analysis in the article "*Chance and Strategy in Poker*" of Laure Elie and Romuald Elie, of the University of Paris. They build upon Alon's analysis extending the basic game to multiple round play, with pre-Flop, Flop, Turn, and River rounds, which follows the format of Texas Hold'Em itself. They also consider challengers who employ a range of strategies. Much of their article is devoted to developing the multiple-round game and calculating the optimal strategy for Alice. Since the game is too complex to calculate the expectation and variance of the each player's winnings, they instead simulate millions of rounds using the computer. This method, appropriately called "Monte Carlo" in the statistics literature, is an extremely effective way to approximate (to a desired level of accuracy) difficult to calculate probabilities, averages and variances.

The analysis presented in this article examines poker games involving blinds, increasing stakes and tournaments (i.e. "knock-out" games). In each, the optimal (or nearly optimal) player Alice is challenged by a range of opponents indexed by their probability p of calling/betting in a given round. The main conclusions are as follows:

¹ *On the other hand, when two equally skilled players challenge each other the outcome is determined predominantly due to chance. This is true for all games, including athletic competitions. This is why a poker match involving the world's best players seems to be often decided by chance.*

- In the basic game, after only 50 rounds of play Alice has at worst less than a 20% chance of being behind even the most skilled challenger ($p=1$). After 500 rounds, this chance is less about $\frac{1}{4}$ of 1%.
- In a game with increasing stakes, Alice has at worst a 12% chance of behind after 50 rounds even against her most skilled challenger ($p= \frac{1}{2}$). After 100 rounds the chance is less than 5%.
- In tournament style play, Alice has less than a 1/100 of 1% chance of being knocked out when each player starts with 100 chips. That chance increases to at most 32% when the starting stakes are only 10 chips.
- In a 4 player game, Alice has less than 20% chance of being behind after only 50 games even against 3 modestly skilled opponents ($p=1$). After 500 plays that chance is less than 1/10 of 1%.

The conclusion is very obvious. A skilled player will trounce lesser skilled opponents not only in the long-run, but also, with high probability, after what amounts to a session of only a couple of hours. Furthermore, in tournament play, where the number of rounds is not fixed, the skilled player has a decisive advantage even with modest initial stakes. **Skill is the dominant and decisive factor.**

V. Summary and Conclusion

Poker while simple enough to learn and play with only a short lesson, is extremely intricate and complex. A skilled player who is able to calculate correctly the probabilities of different hand configurations and is able to use that knowledge to bet and bluff appropriately has a substantial advantage over players without these skills. The two papers evaluated here ably demonstrate using mathematical analysis and computer simulation exactly how decisive that advantage is. The player who just “hopes to get the cards” will get them from time to time, but even after a single evening of play against a top player, he will be decisively beaten. **Skill dominates chance in poker.**